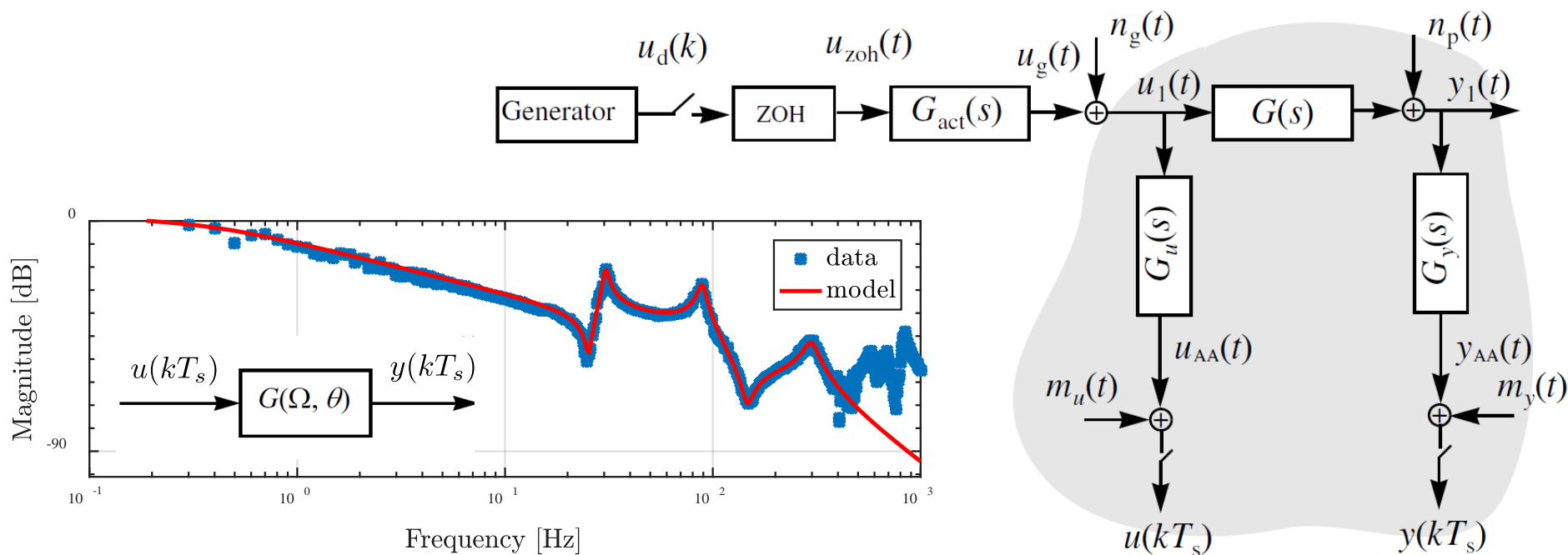


FREQUENCY-DOMAIN SYSTEM IDENTIFICATION FOR HIGH-PRECISION CONTROL DESIGN

Thomas Beauduin, Hiroshi Fujimoto (The University of Tokyo)



1.1 Main References

- [1] K. Astrom, *Introduction to Stochastic Control Theory*. Academic Press: New York, 1970.
- [2] G. Box and G. Jenkins, *Time Series Analysis: Forecasting and control*. Holden-Day: Oakland, 1970.
- [3] G. Goodwin and R. Payne, *Dynamic System Identification. Experiment Design and Data Analysis*. Academic Press: New York, 1977.
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- [8] L. Ljung, *System Identification: theory for the user*. MIT Press, Cambridge, MA, 1999.
- [9] R. Pintelon and J. Schoukens, *Identification of linear systems: a practical guideline to accurate modeling*. Pergamon Oxford, 2001.
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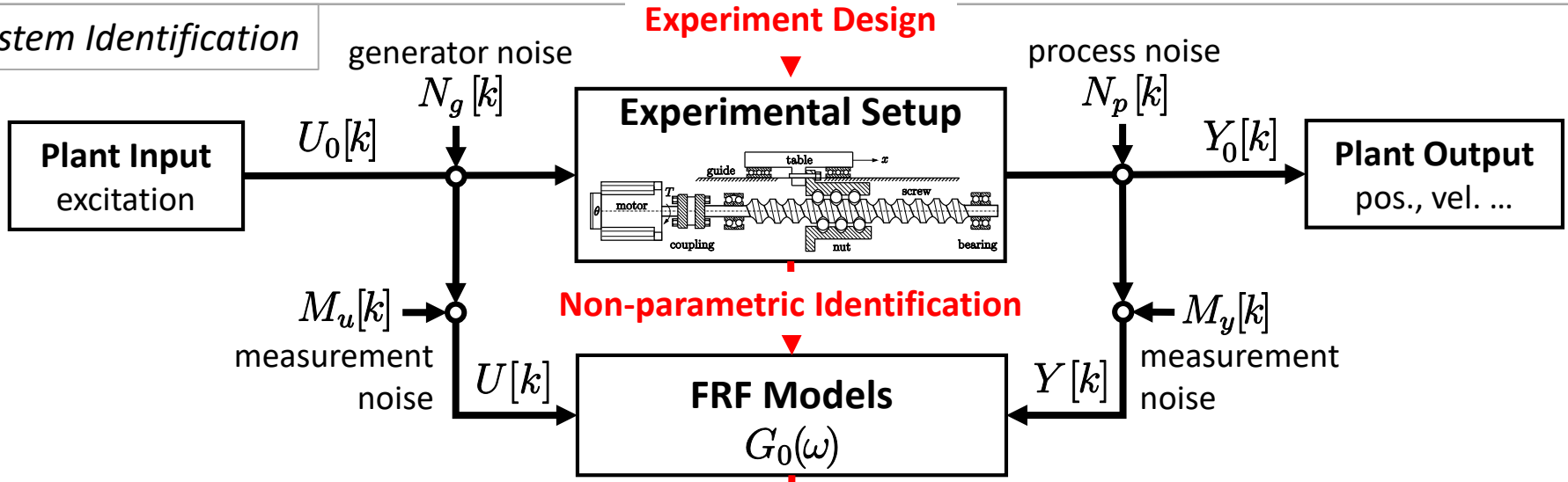
1.2 INTRODUCTION

System Identification for Control

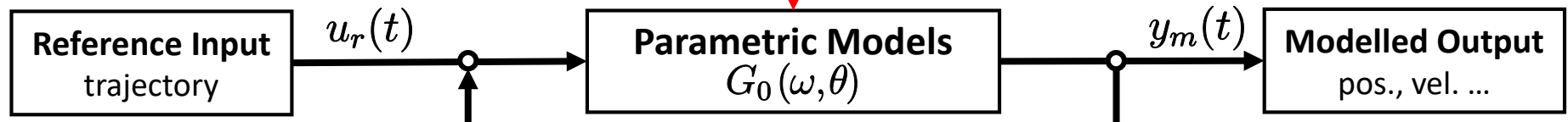
System Modelling



System Identification



Parametric Identification



System Control



ω angular frequency
 k discrete frequencies
 θ finit parameter vector

2.1 NON-PARAMETRIC Overview of Non-Parametric Methods

• Methods

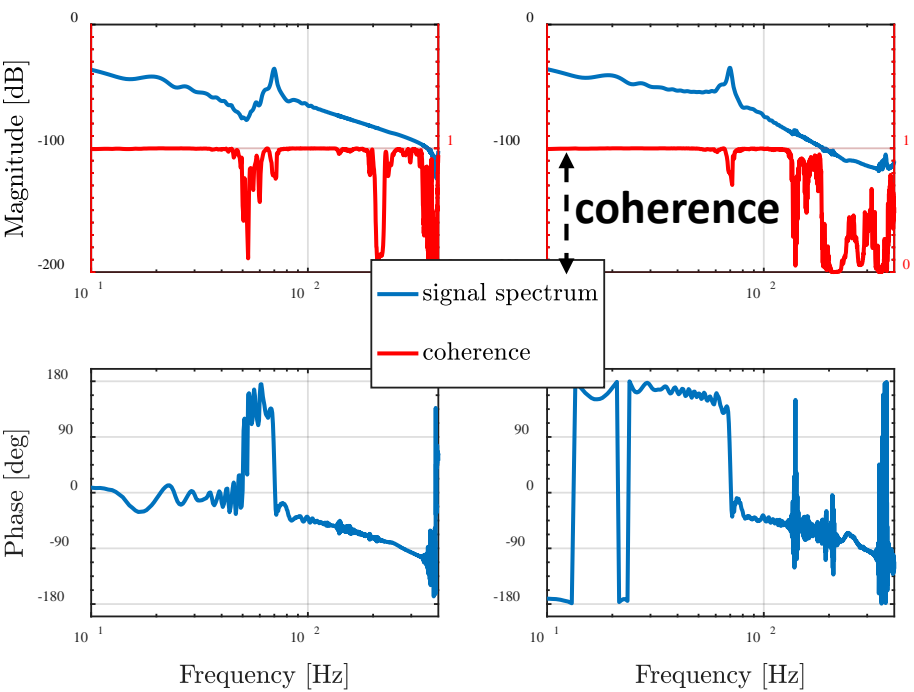
Servo Analyzer

Random excitation
Output-only noise

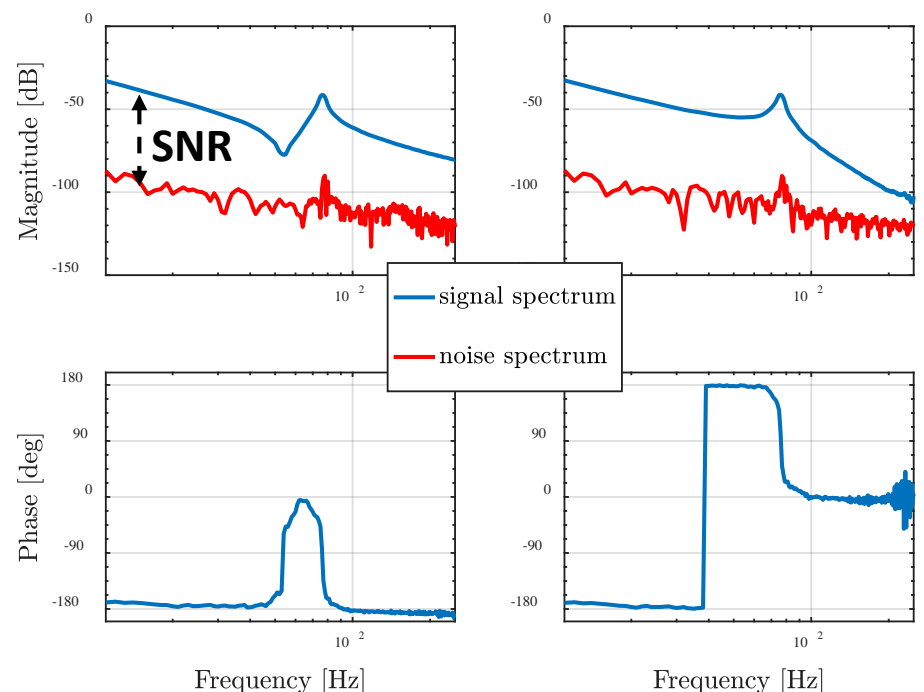
Proposed Analyzer

Periodic excitation
Input-output noise

• Results



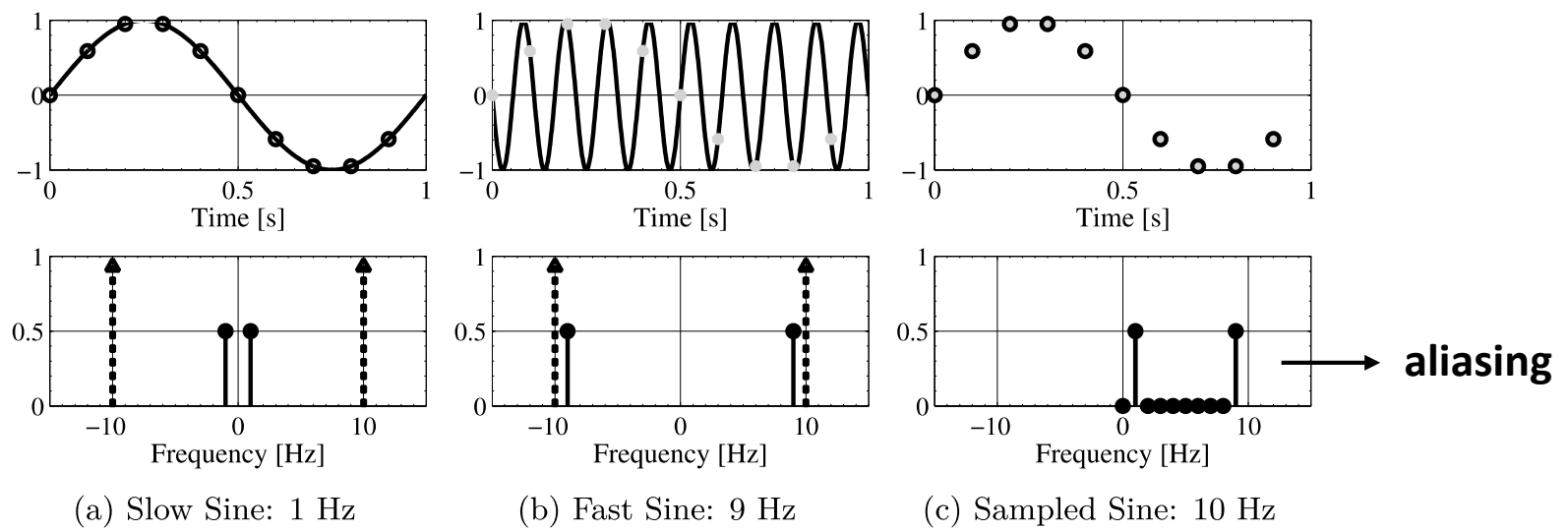
2-5min experiment



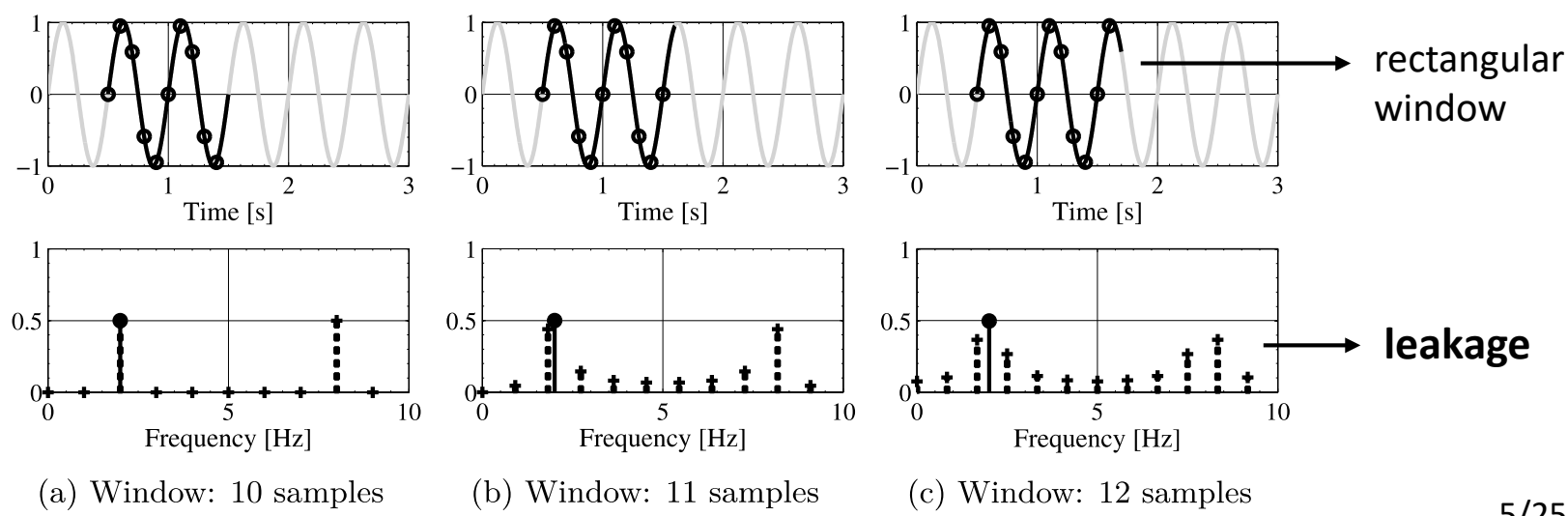
10s experiment

2.2 NON-PARAMETRIC Discrete Fourier Transform – Problems

- Aliasing errors: high frequency components mirrored at lower frequencies around Nyquist

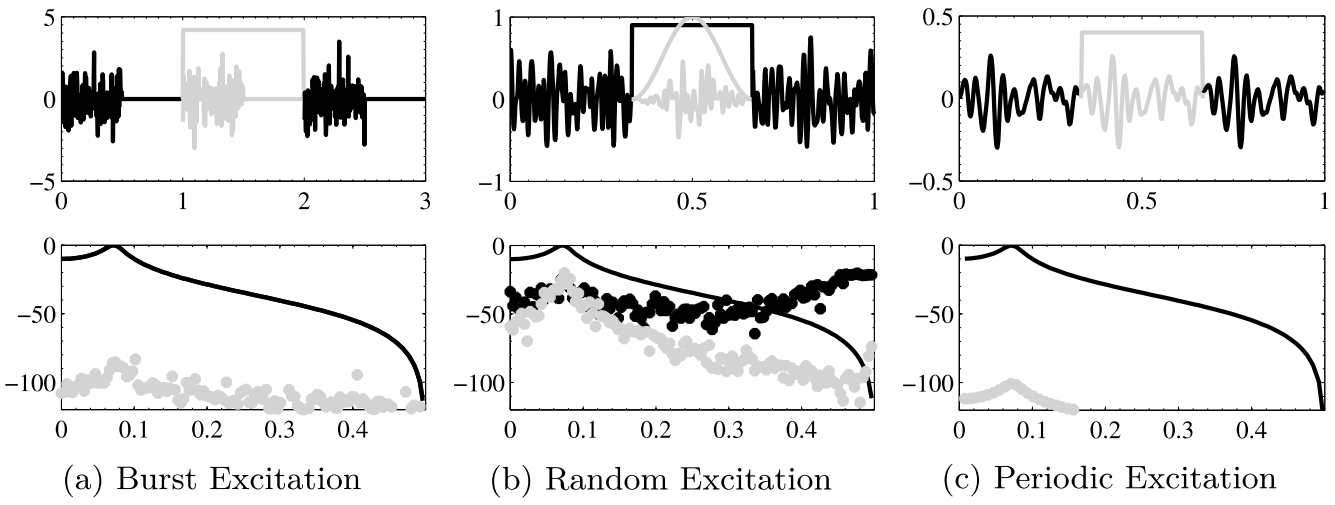


- Leakage errors: signal spectrum smeared out due to finite measurement length

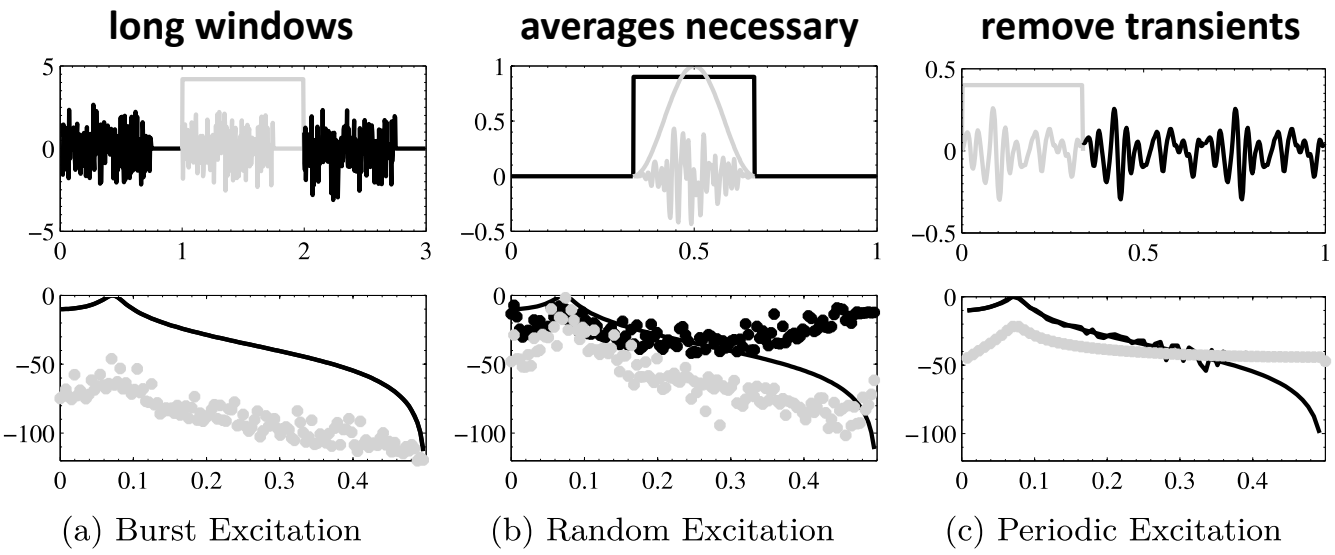


2.2 NON-PARAMETRIC Discrete Fourier Transform – Remedies

- **Anti-Aliasing:** set high sampling frequencies, don't excite near-Nyquist, anti-aliasing filters
- **Anti-Leakage:** 2 solutions & 1 trade-off



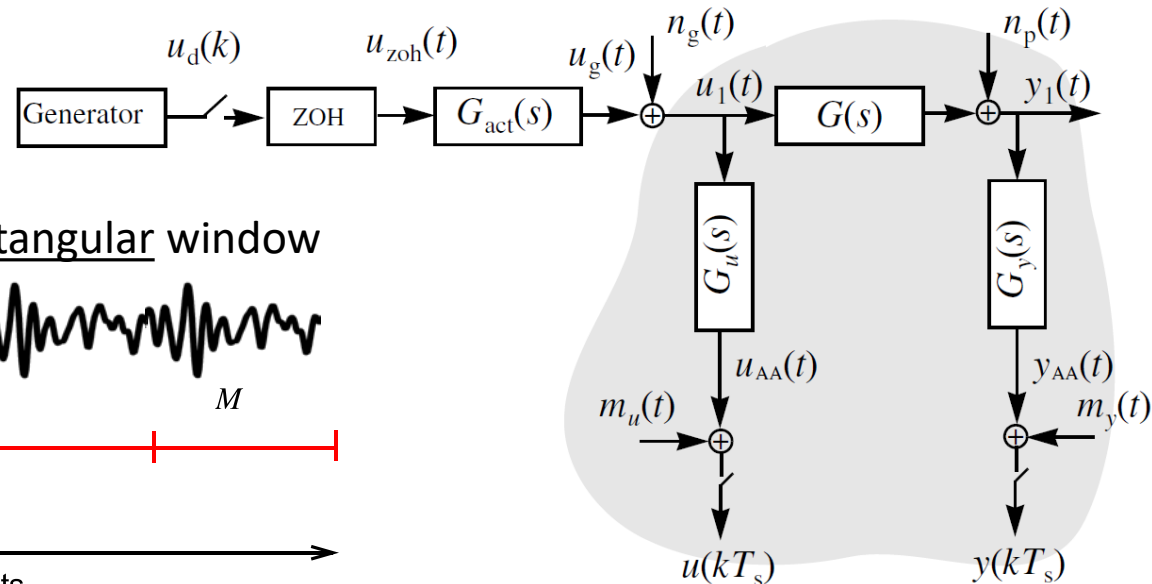
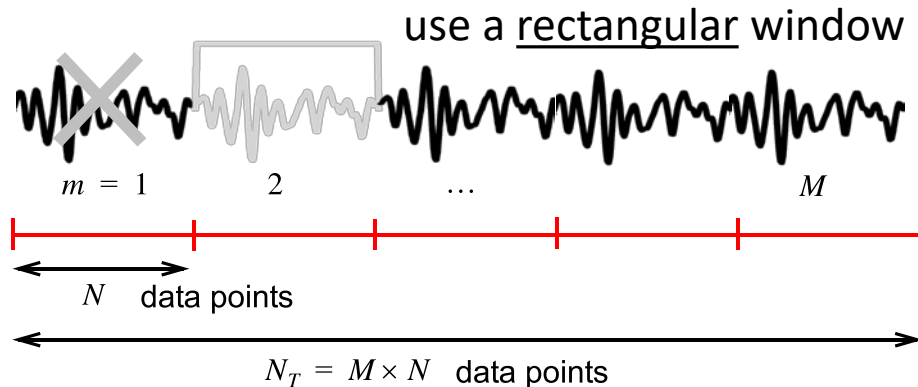
• **Pitfalls:**



2.3 NON-PARAMETRIC Periodic Excitation – Acquisition

• Procedure

measure M consecutive periods of steady state response



• Data Obtained

Sample means

$$\hat{Y}(k) = \frac{1}{M} \sum_{m=1}^M Y^{(m)}(k), \quad \hat{U}(k) = \frac{1}{M} \sum_{m=1}^M U^{(m)}(k)$$

Sample (co-)variances

$$\hat{\sigma}_Y^2(k) = \frac{1}{M(M-1)} \sum_{m=1}^M |Y^{(m)}(k) - \hat{Y}(k)|^2, \quad \hat{\sigma}_U^2(k) = \frac{1}{M(M-1)} \sum_{m=1}^M |U^{(m)}(k) - \hat{U}(k)|^2$$

$$\hat{\sigma}_{YU}^2(k) = \frac{1}{M(M-1)} \sum_{m=1}^M (Y^{(m)}(k) - \hat{Y}(k))(U^{(m)}(k) - \hat{U}(k))$$

Note : decrease FRF estimate uncertainty = M larger for given N_T
 increasing FRF frequency resolution = N larger for given N_T

2.3 NON-PARAMETRIC Periodic Excitation – Processing

Frequency Response

Maximum likelihood approach assuming noise on both records

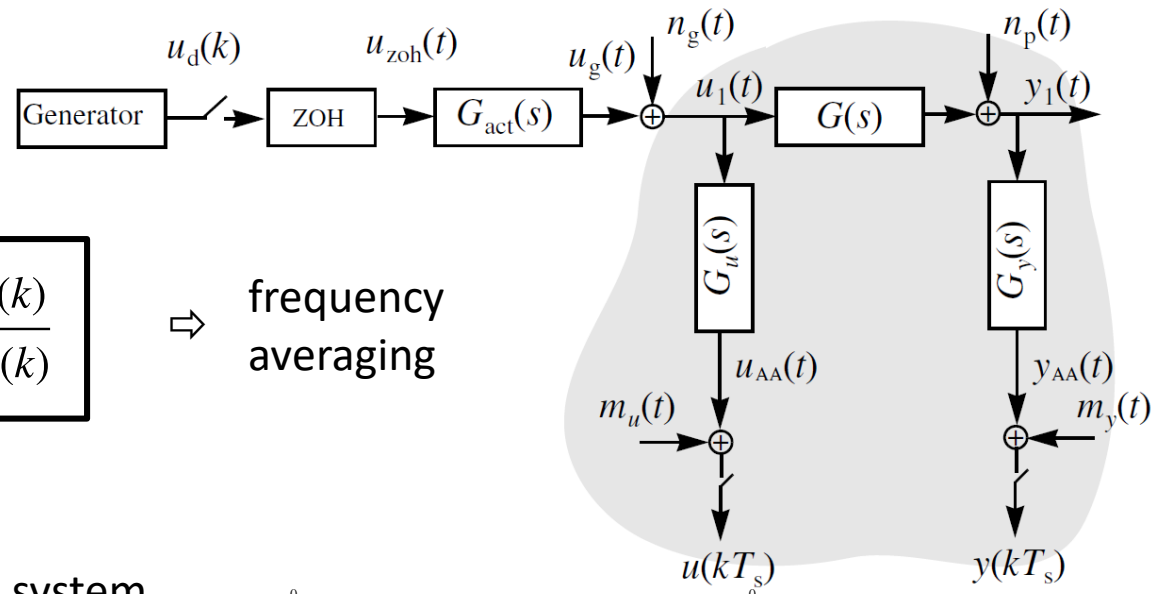
$$\hat{G}_{ML}(j\omega_k) = \frac{\hat{Y}(k)}{\hat{U}(k)} = \frac{M^{-1} \sum_{m=1}^M Y^{(m)}(k)}{M^{-1} \sum_{m=1}^M U^{(m)}(k)}$$

Variance

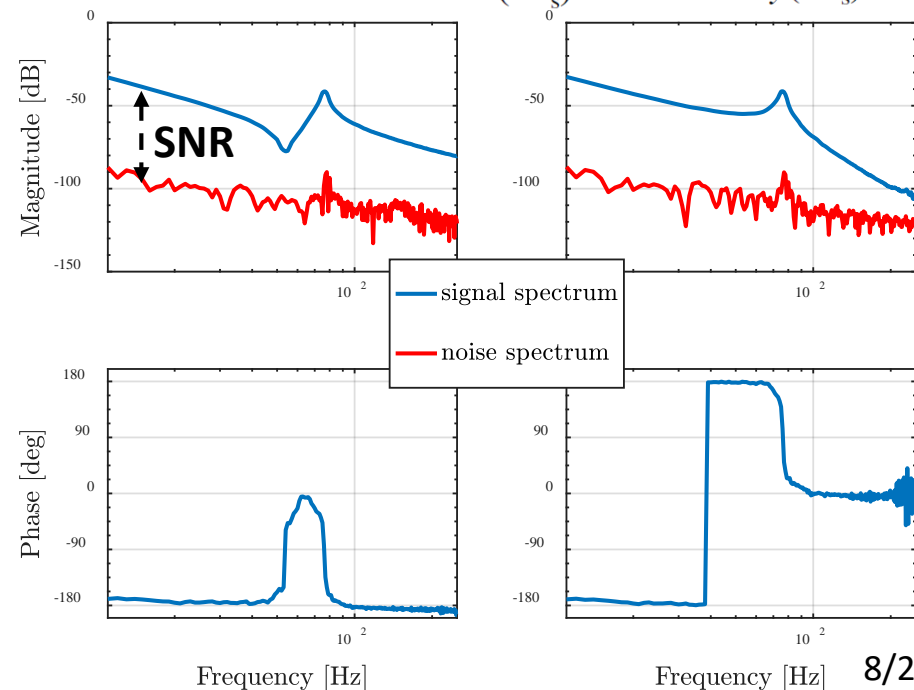
Sample variance of non-parametric system model also called Cramer-Rao lower bound

$$\hat{\sigma}_G^2(k) = |\hat{G}_{ML}(j\omega_k)|^2 \cdot \left[\frac{\hat{\sigma}_Y^2(k)}{|\hat{Y}(k)|^2} + \frac{\hat{\sigma}_U^2(k)}{|\hat{U}(k)|^2} - 2\text{Re} \left(\frac{\hat{\sigma}_{YU}^2(k)}{\hat{Y}(k)\overline{\hat{U}(k)}} \right) \right]$$

⇒ non-parametric noise model



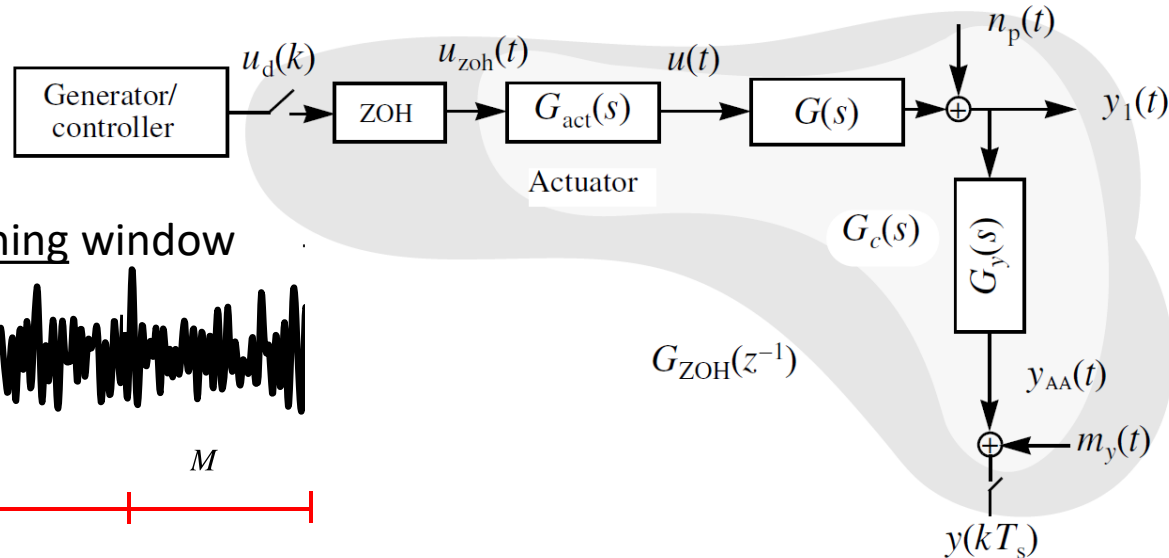
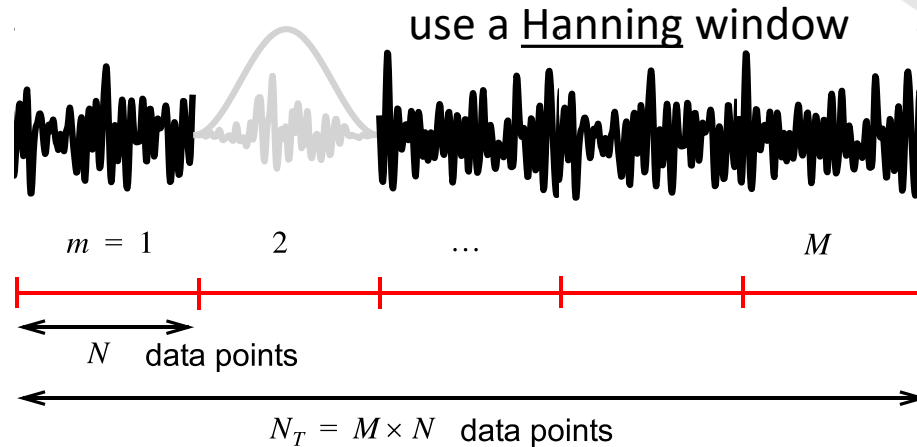
⇒ frequency averaging



2.4 NON-PARAMETRIC Random Excitation – Acquisition

• Procedure

Divide the input-output records in M segments



• Data Extraction

Sample auto- and cross-power spectra

$$\hat{S}_{Y_w}(k) = M^{-1} \sum_{m=1}^M |Y_w^{(m)}|^2, \quad \hat{S}_{U_w}(k) = M^{-1} \sum_{m=1}^M |U_w^{(m)}|^2$$

$$\hat{S}_{Y_w U_w}(k) = M^{-1} \sum_{m=1}^M Y_w^{(m)}(k) \overline{U_w^{(m)}(k)}$$

Sample variance

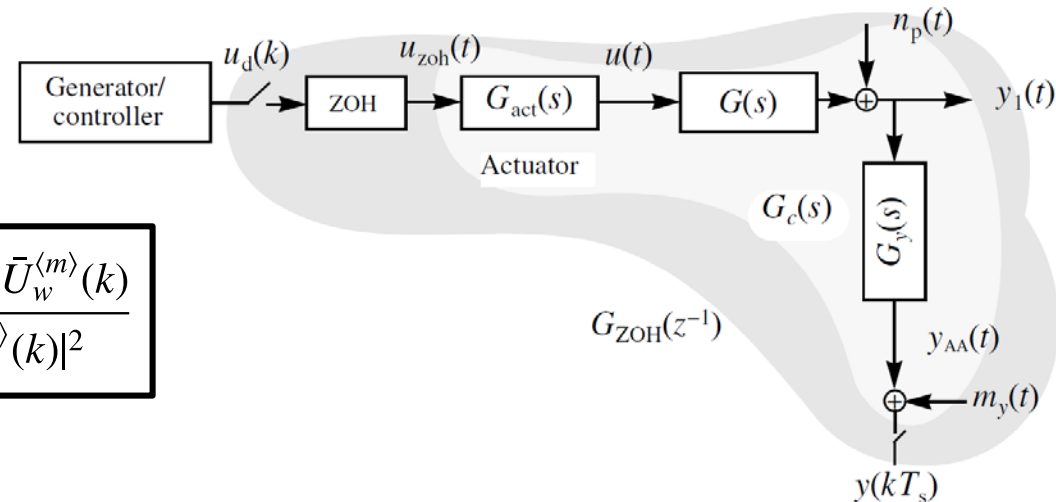
$$\hat{\sigma}_{\hat{G}}^2(k) = |\hat{G}(j\omega_k)|^2 \frac{1 - \gamma^2(\omega_k)}{\gamma^2(\omega_k)}, \quad \text{with} \quad \gamma^2(\omega_k) = \frac{|\hat{S}_{Y_w U_w}(j\omega_k)|^2}{\hat{S}_{U_w}(j\omega) \hat{S}_{Y_w}(j\omega_k)}$$

2.4 NON-PARAMETRIC Random Excitation – Processing

• Frequency Response

Least squares approach assuming input signal is exactly known

$$\hat{G}_1(j\omega_k) = \frac{\hat{S}_{Y_w U_w}(k)}{\hat{S}_{U_w}(k)} = \frac{M^{-1} \sum_{m=1}^M Y_w^{(m)}(k) \bar{U}_w^{(m)}(k)}{M^{-1} \sum_{m=1}^M |U_w^{(m)}(k)|^2}$$



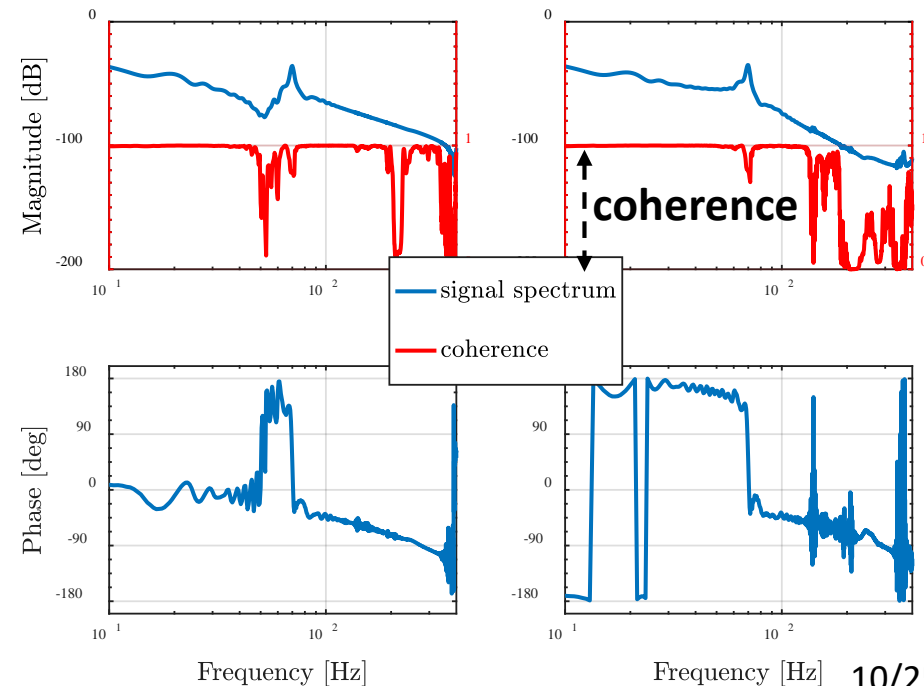
• Coherence

Quantifies the quality of data by measuring the input to output linear relationship

$$\gamma^2(\omega_k) = \frac{|\hat{S}_{Y_w U_w}(j\omega_k)|^2}{\hat{S}_{U_w}(j\omega) \hat{S}_{Y_w}(j\omega_k)} \quad 0 \leq \gamma^2(\omega_k) \leq 1$$

Note : coherences indicates presence of

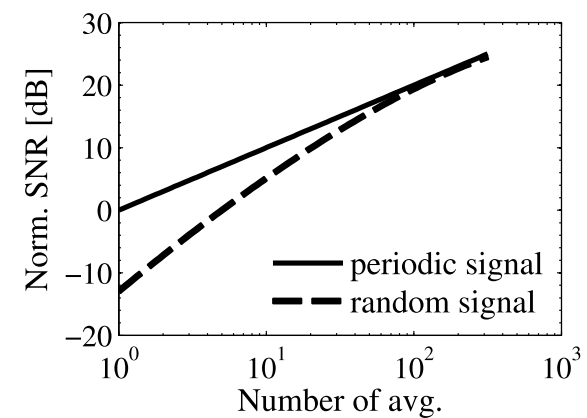
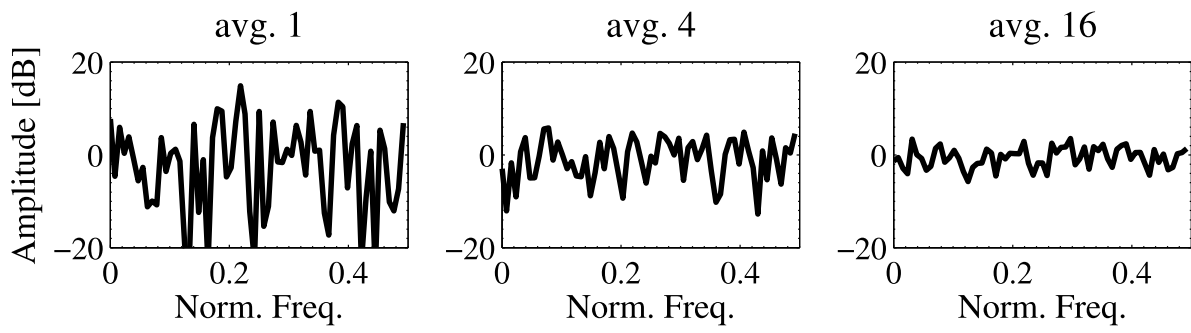
- extraneous noise
- non-linear distortions
- leakage errors
- non-measured input or disturbance



2.5 NON-PARAMETRIC Problems with Random Excitations

- **Longer experiment time**

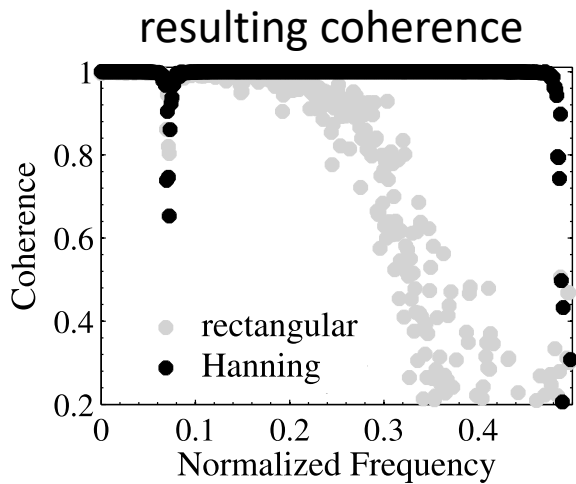
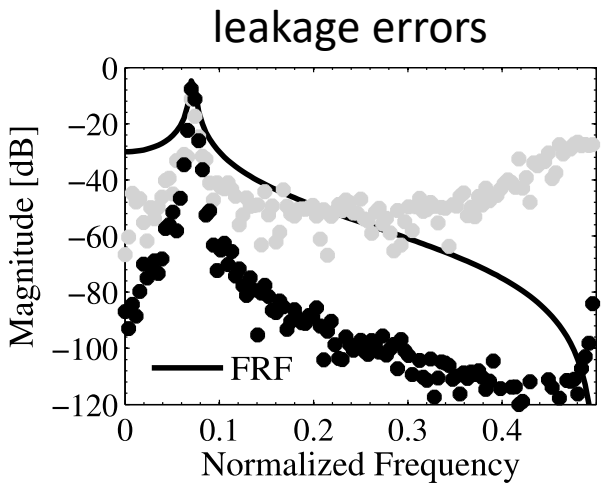
white noise input amplitude spectrum contains random large drops



Input power spectrum of white noise sequences averaged over M -blocks

- **Lower FRF resolution**

windowing reduces leakage, but not in strong varying FRF frequencies, e.g. (anti-)resonances



3.1 Experiment Design

• Constrained optimization

minimize $\max_{k=1\dots F} (\sigma_G^2(k))$ \longrightarrow i.e. minimize peak variance

subject to amplitude constraint: $\max_{t \in [0, t_f]} |u(t)| \leq u_{max}$ \longrightarrow e.g. max stage stroke

input power constraint: $\int_{t_0}^{t_f} |u(t)|^2 dt \leq P_{max}$ \longrightarrow e.g. max actuator power

acquisition constraints: $t_f \leq t_{max}, f_s \leq f_{s,max}$ \longrightarrow e.g. embedded system

• Non-parametric design criteria

minimal and flat SNR i.e. define two characteristics:

Crest factor $Cr(u) = \frac{u_{peak}}{u_{rmse}} = \frac{\max_{t \in N} |u(t)|}{u_{rms} \sqrt{P_{int}/P_{tot}}}$ \longrightarrow i.e. signal compactness

Time factor * $Tf(u) = \max_{k \in F} 0.5 Cr^2(u) \frac{U_{rmse}^2}{|U(k)|^2}$ \longrightarrow i.e. efficiency of signal

(*) required measurement time per frequency line to reach desired SNR

3.2

EXCITATION

Excitation Signals Comparison

Random

Random noise

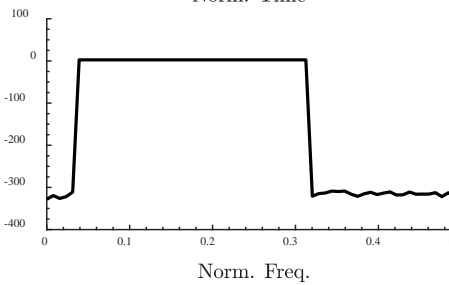
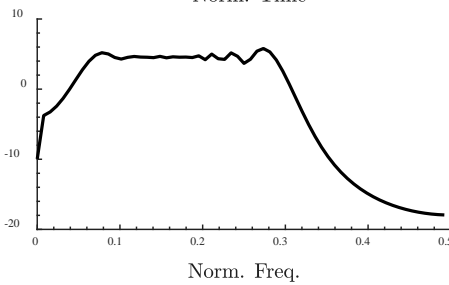
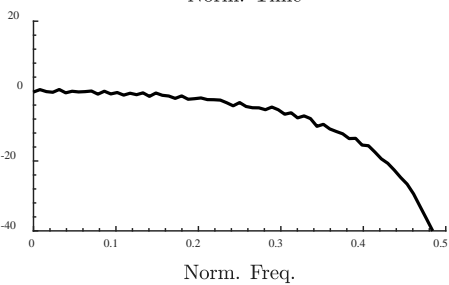
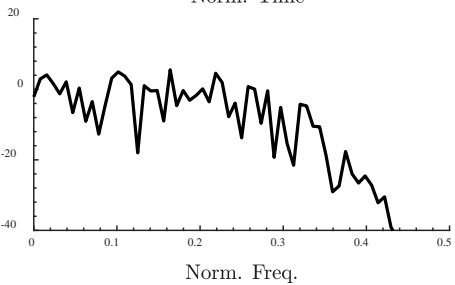
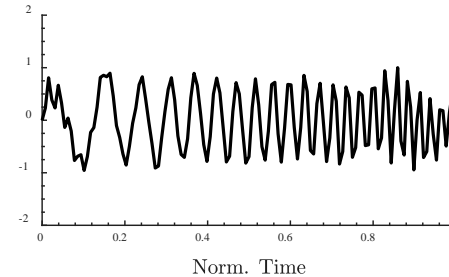
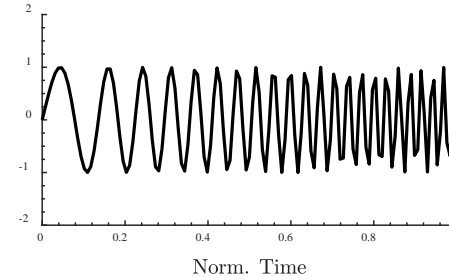
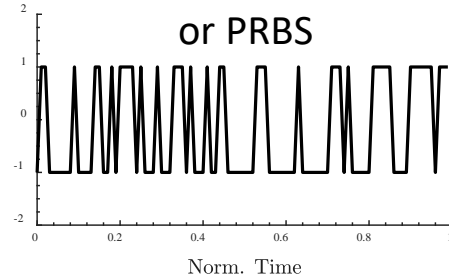
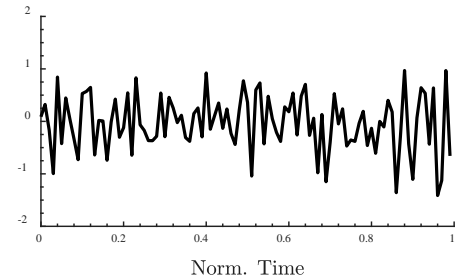
Binary sequence
or PRBS

Periodic

Periodic

Sweptsine

Multisine



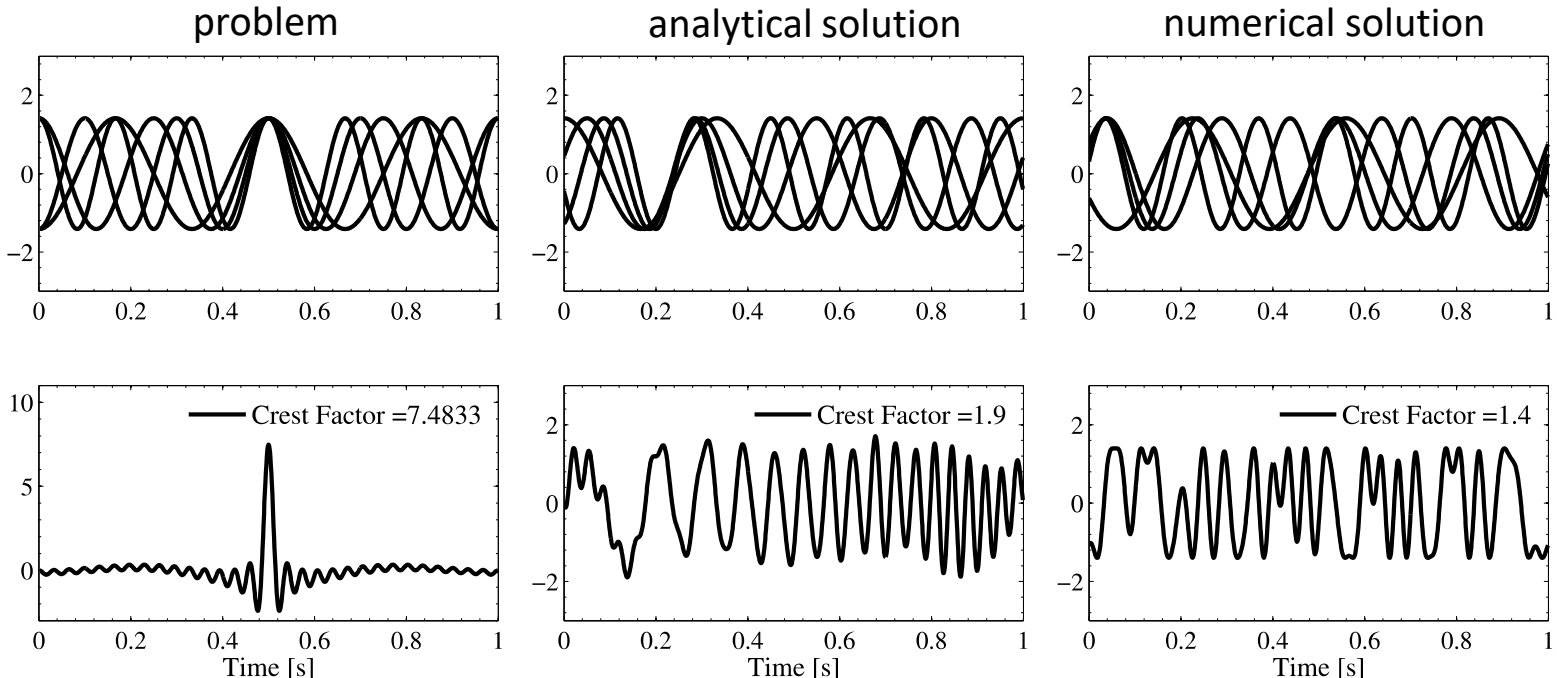
Type	Signal	Crest Factor	Time Factor	Leakage	Arbitrary Amplitude Spectrum
Periodic	Swept Sine	1.45	1.5 - 4	no	no
	Stepped Sine	1.41	1 ^(*)	no	yes
	Multisine	1.41 - 1.7	1	no	yes
Random	PRBS	1	1.5 - 4.5	no	no
	Random Noise	3	4.5	yes	no

(*) without considering transients between steps

3.3 EXCITATION DESIGN

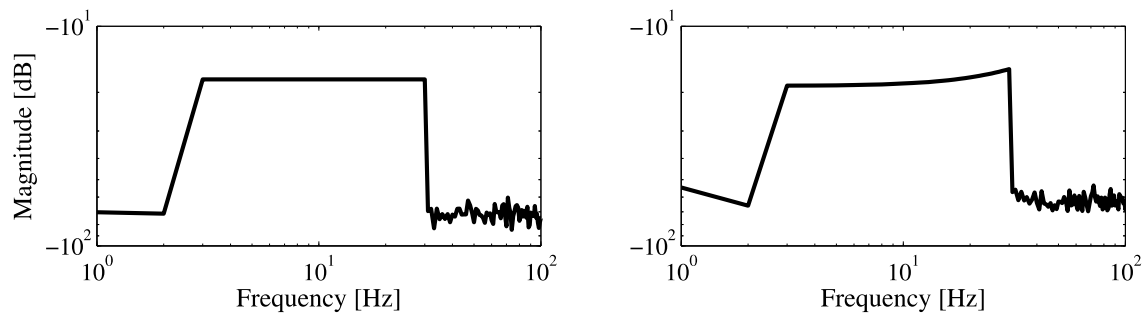
Multisine Optimization

- Crest Factor optimization** – Phase calculation



(a) Fixed phase (b) Schroeder phase (c) Optimized phase

- Time Factor optimization** – Amplitude spectrum shaping for flat SNR



(*) example:
see beyond the
current loop
low-pass filter

4.1

PARAMETRIC Parametric Methods Overview

TDI

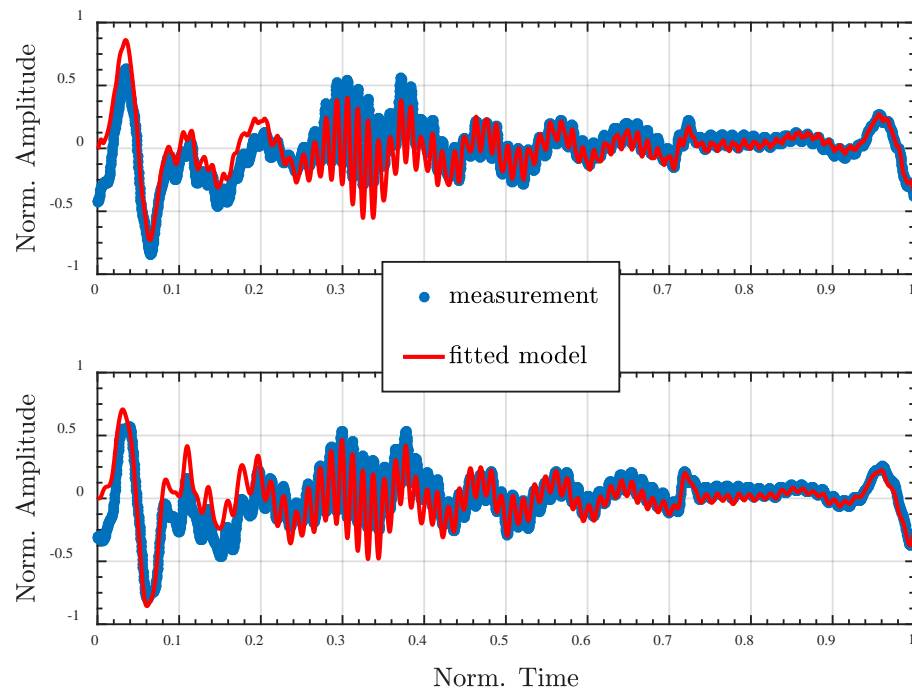
with random excitation

- Discrete-time modeling
- Minimum-Phase zero's only
- Parametric noise model

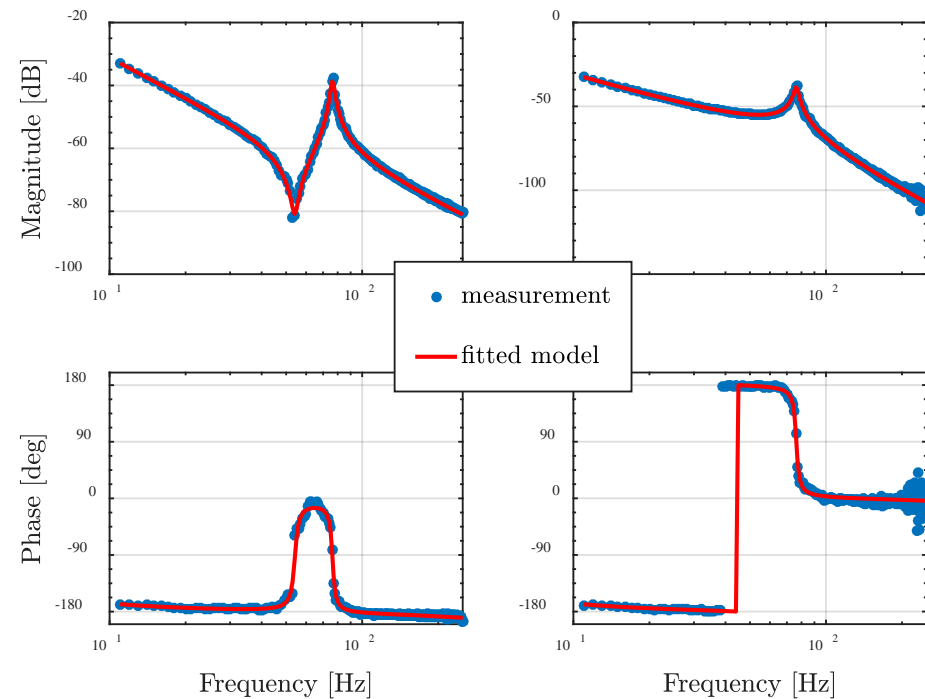
FDI

with periodic excitation

- Continuous-time modeling
- Non-Minimum-Phase zero's possible
- Non-parametric noise model



5.24s – SISO optimization
Conventional



0.32s – SIMO optimization
Proposed

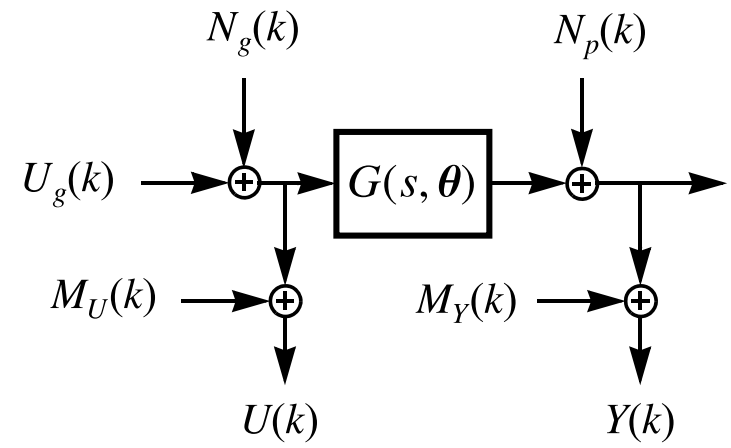
4.2 PARAMETRIC Frequency Domain Identification (FDI)

- Parametrization:** Error-In-Variables structure

- $N_g(k)$ generator noise
- $N_p(k)$ process noise
- $M_U(k)$ input measurement noise
- $M_Y(k)$ output measurement noise

$$G(s, \theta) = \frac{N(s, \theta)}{D(s, \theta)} = \frac{\sum_{r=0}^{n_b} b_{n_b-r} s^r}{\sum_{r=0}^{n_a} a_{n_a-r} s^r}$$

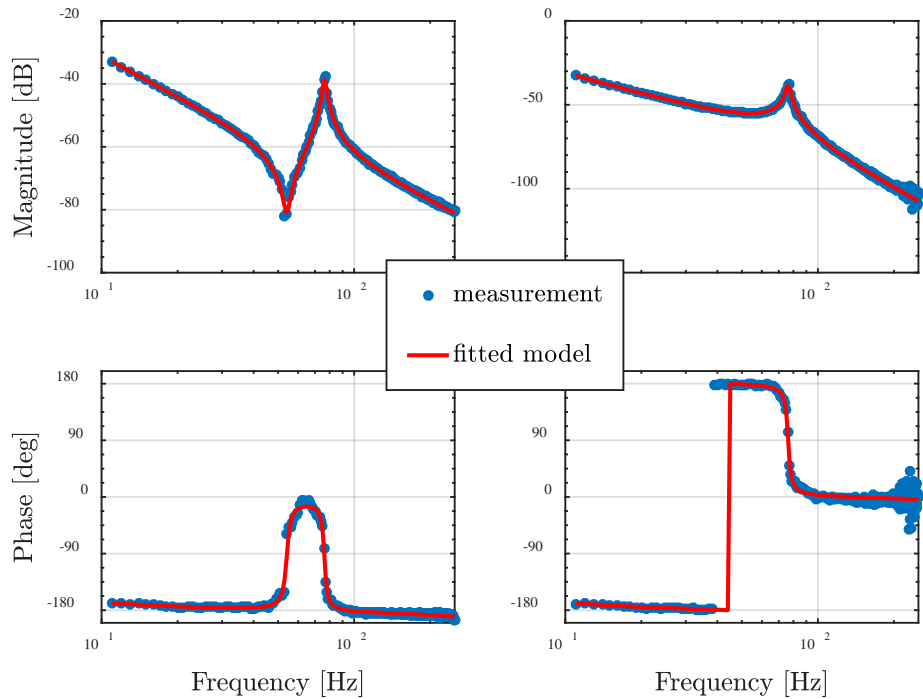
with $\theta = [a_0 \ \dots \ a_{n_a} \ b_0 \ \dots \ b_{n_b}]^T$



- Optimization:** non-linear least squares

$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| \underbrace{W(s_k)}_{\text{green}} \left(\underbrace{\hat{G}(s_k)}_{\text{blue}} - \underbrace{G(s_k, \theta)}_{\text{red}} \right) \right|^2$$

$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| \underbrace{\frac{W(s_k)}{D(s_k, \theta)}}_{\text{red}} \left[\underbrace{\hat{Y}(s_k)}_{\text{blue}} - \underbrace{\hat{U}(s_k)}_{\text{blue}} \right] \left[\underbrace{\frac{D(s_k, \theta)}{N(s_k, \theta)}}_{\text{red}} \right] \right|^2$$



4.2 PARAMETRIC FD Identification Algorithm

- **Problem Formulation: Non-linear Least Squares**

$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| \frac{W(s_k)}{D(s_k, \theta)} \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta) \\ N(s_k, \theta) \end{bmatrix} \right|^2$$

- **Problem Reformulation**

Approach 1: Analytical Linear Least Squares

$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| W(s_k) \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta) \\ N(s_k, \theta) \end{bmatrix} \right|^2 \longrightarrow \text{biased, but useful for starting-values}$$

Approach 2: Iteratively Linear Least Squares

a) Sanathan-Koerner:
$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| \frac{W(s_k)}{D(s_k, \theta^{(i-1)})} \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta^{(i)}) \\ N(s_k, \theta^{(i)}) \end{bmatrix} \right|^2$$

b) Gauss-Newton:
$$J_{GN}^{(i)} = \frac{\partial V_{NL}^{(i)}}{\partial \theta} = 0$$

$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| \frac{W(s_k)}{D(s_k, \theta^{(i-1)})} \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \left[N(s_k, \theta^{(i-1)}) - \frac{D(s_k, \theta^{(i-1)})}{D(s_k, \theta^{(i-1)})} \frac{N(s_k, \theta^{(i-1)})D(s_k, \theta^{(i)})}{D(s_k, \theta^{(i-1)})} + N(s_k, \theta^{(i)}) \right] \right|^2$$

- c) Instrumental Variable:

$$\sum_{k=1}^F \left[\frac{-\partial \hat{P}(s_k, \theta^{(i-1)})}{\partial \theta^T} \right]^* W^*(s_k) \frac{W(s_k)}{D(s_k, \theta^{(i-1)})} \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta^{(i)}) \\ N(s_k, \theta^{(i)}) \end{bmatrix} = 0$$

FD Identification Weighting

• General Formulation

$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| \frac{\overline{W(s_k)}}{D(s_k, \theta)} \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta) \\ N(s_k, \theta) \end{bmatrix} \right|^2$$

Deterministic Approach

$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| \frac{w_k}{D(s_k, \theta)} \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta) \\ N(s_k, \theta) \end{bmatrix} \right|^2$$

with $w_k \simeq \gamma^2(\omega_k)$

no a-priori information used

Weighted Least Squares (Markov) Estimator

$$\min_{\theta^{(i)}} \sum_{k=1}^F \frac{|\varepsilon(s_k, \theta, Z(k))|^2}{\underline{w_k^{-1}}}$$

Stochastic Approach

$$\min_{\theta^{(i)}} \sum_{k=1}^F \left| \frac{\hat{C}^{-1}(s_k)}{D(s_k, \theta)} \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta) \\ N(s_k, \theta) \end{bmatrix} \right|^2$$

with $\hat{C}(s_k) = \begin{bmatrix} \hat{\sigma}_U^2(s_k) & \hat{\sigma}_{YU}^2(s_k) \\ \hat{\sigma}_{YU}^2(s_k) & \hat{\sigma}_Y^2(s_k) \end{bmatrix}$

Sample (co-)variances used

Maximum Likelihood Estimator

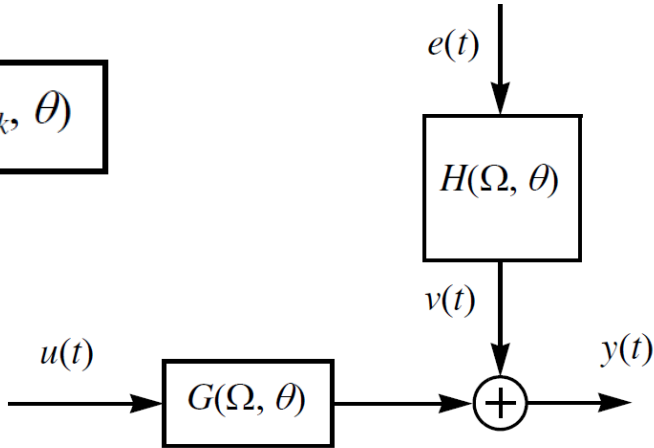
$$\min_{\theta^{(i)}} \sum_{k=1}^F \frac{|\varepsilon(s_k, \theta, Z(k))|^2}{\underline{\sigma_\varepsilon^2(s_k, \theta)}}$$

$$\min_{\theta^{(i)}} \sum_{k=1}^F \frac{\left| \begin{bmatrix} \hat{Y}(s_k) & -\hat{U}(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta) \\ N(s_k, \theta) \end{bmatrix} \right|^2}{\sigma_Y^2(s_k)|D(s_k, \theta)|^2 + \sigma_U^2(s_k)|N(s_k, \theta)|^2 - 2\text{Re}(\sigma_{YU}^2(s_k)D(s_k, \theta)\overline{N}(s_k, \theta))}$$

4.3 PARAMETRIC Time-Domain Identification (TDI)

- **Parametrization:** parametric noise structure

$$Y(k) = G(\Omega_k, \theta)U(k) + T_G(\Omega_k, \theta) + H(\Omega_k, \theta)E(k) + T_H(\Omega_k, \theta)$$



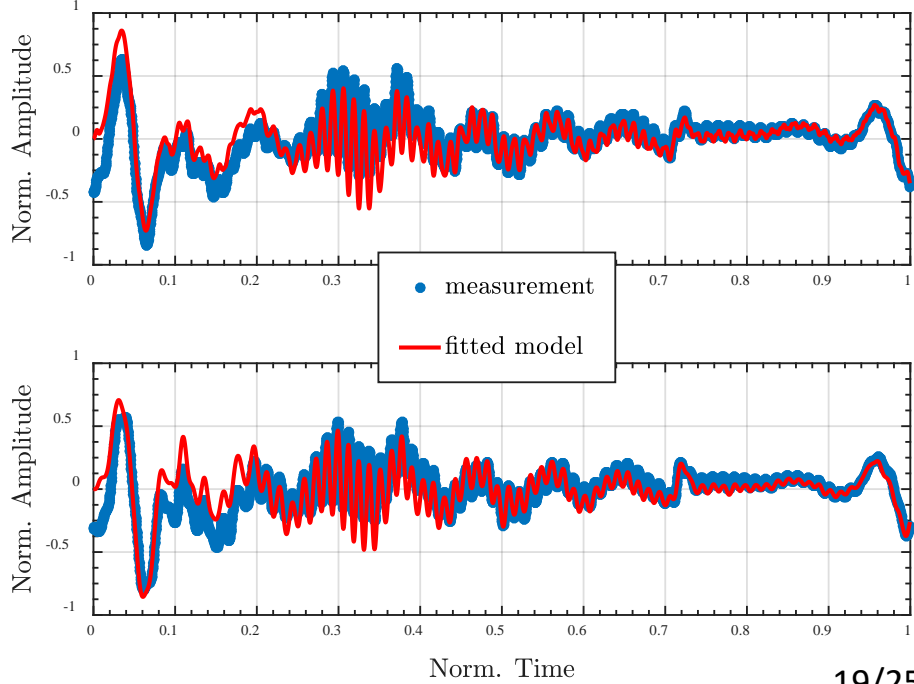
	Plant
model	$G(\Omega, \theta) = \frac{B(\Omega, \theta)}{A(\Omega, \theta)}$
transient	$T_G(\Omega, \theta) = \frac{I_G(\Omega, \theta)}{A(\Omega, \theta)}$

	Noise
	$H(\Omega_k, \theta) = \frac{C(\Omega, \theta)}{D(\Omega, \theta)}$
	$T_H(\Omega, \theta) = \frac{I_H(\Omega, \theta)}{D(\Omega, \theta)}$

- **Model structures:**

AR = autoregressive
 MA = moving average
 X = exogenous input
 BJ = Box-Jenkins

ARX: $C = 1, D = A, T_H = 0$
 ARMAX: $D = A, T_H = 0, n_{i_g} \geq \max(n_a, n_b, n_c) - 1$
 ARMA: $G = 0, T_G = 0$
 BJ: $D \neq A$



4.3 PARAMETRIC Time-Domain Identification Methods

- **Estimation Framework**

One-step-ahead prediction

$$\begin{aligned}v(t) &= H(q)e(t) = e(t) + (H(q) - 1)e(t) \\ &= e(t) + (1 - H^{-1}(q))v(t)\end{aligned}$$

One-step-ahead predictor output

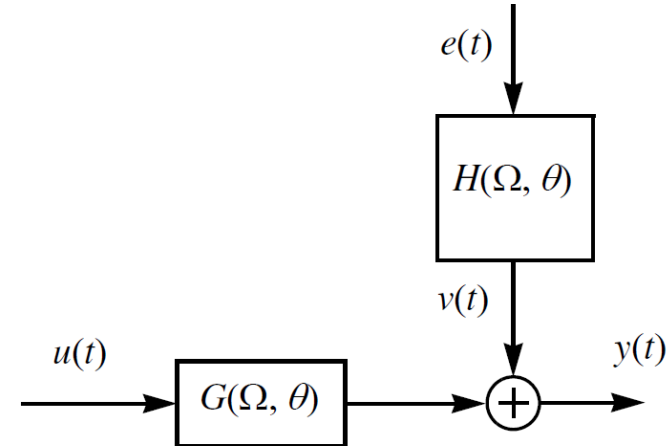
$$\begin{aligned}\hat{y}(t|t-1) &= G(q)u(t) + \hat{v}(t|t-1) \\ &= H^{-1}(q)G(q)u(t) + (1 - H^{-1}(q))y(t)\end{aligned}$$

Prediction error

$$\begin{aligned}\varepsilon(t) &= y(t) - \hat{y}(t|t-1) \\ &= H^{-1}(q) (y(t) - G(q)u(t))\end{aligned}$$

Prediction error cost function

$$V_{PE}(\boldsymbol{\theta}, z) = \sum_{t=0}^{N-1} \varepsilon^2(t, \boldsymbol{\theta}) = \sum_{t=0}^{N-1} \left(H^{-1}(q, \boldsymbol{\theta}) (y(t) - G(q, \boldsymbol{\theta})u(t)) \right)^2$$



5.1

EXPERIMENTS

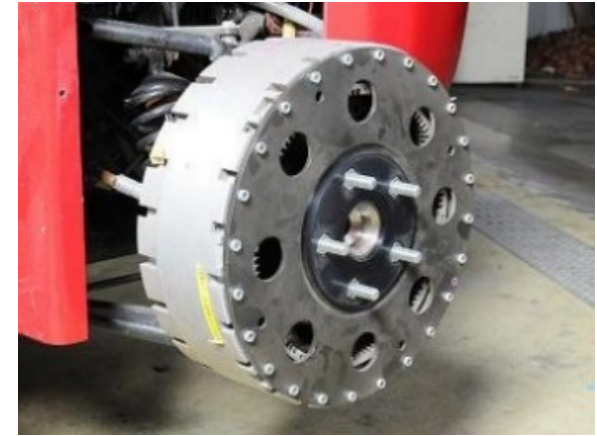
HFLab Application Range



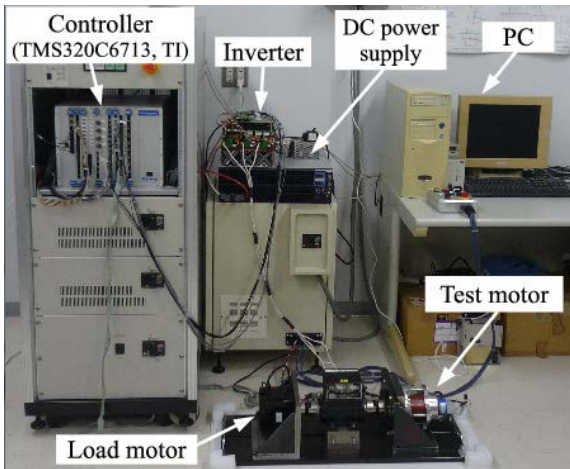
Machine-tool stages



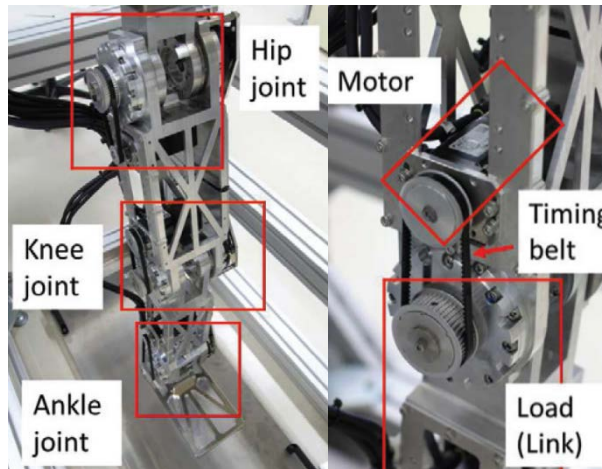
High-precision stages



In-Wheel-Motors



Motors & converters



Robotics

other challenges ...

<http://hflab.k.u-tokyo.ac.jp/>

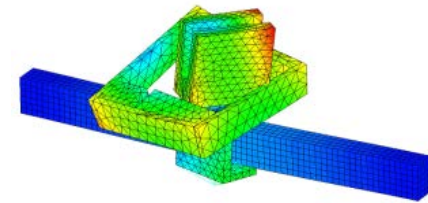
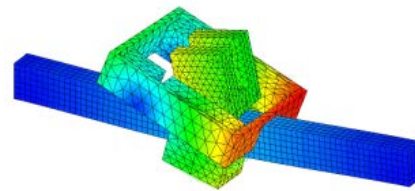
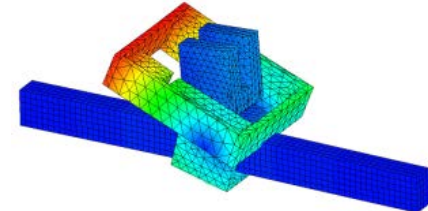
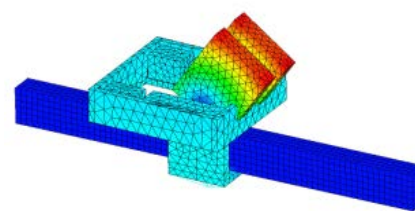
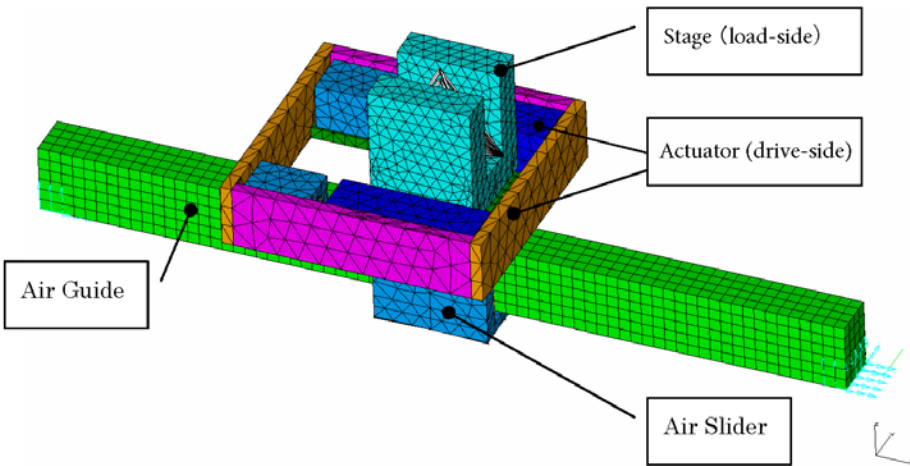
Open Source Matlab Toolbox: www.github.com/thomasbeauduin/FdiTools

5.2

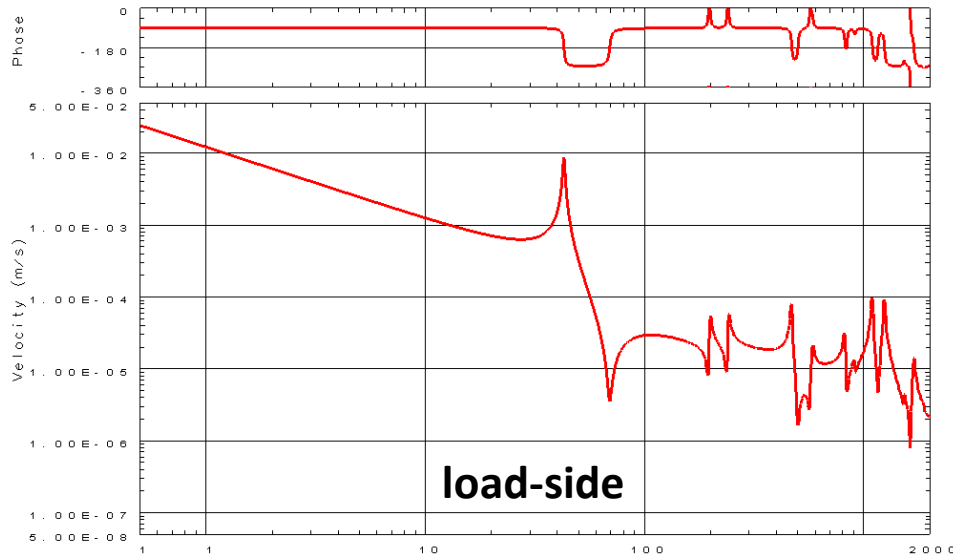
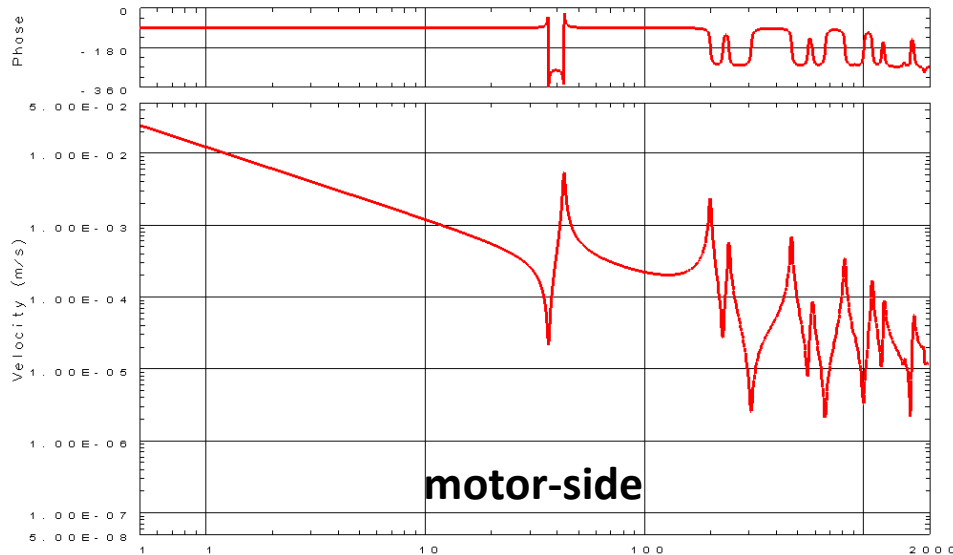
EXPERIMENTS

High-precision Stage – Finite Element Model

- Finite-Element-Analysis



- Bode-diagrams

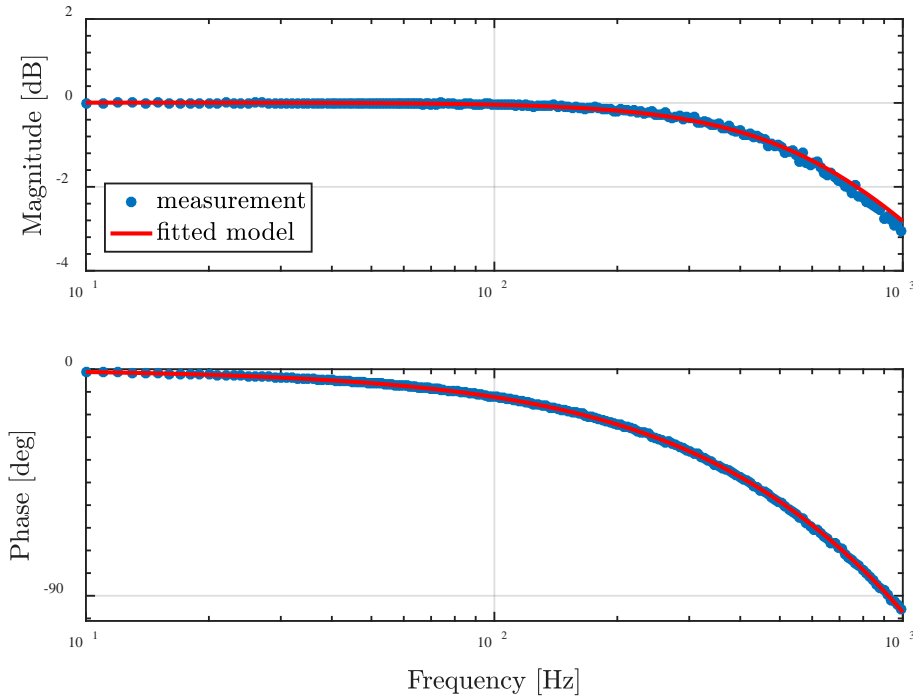


5.3

EXPERIMENTS

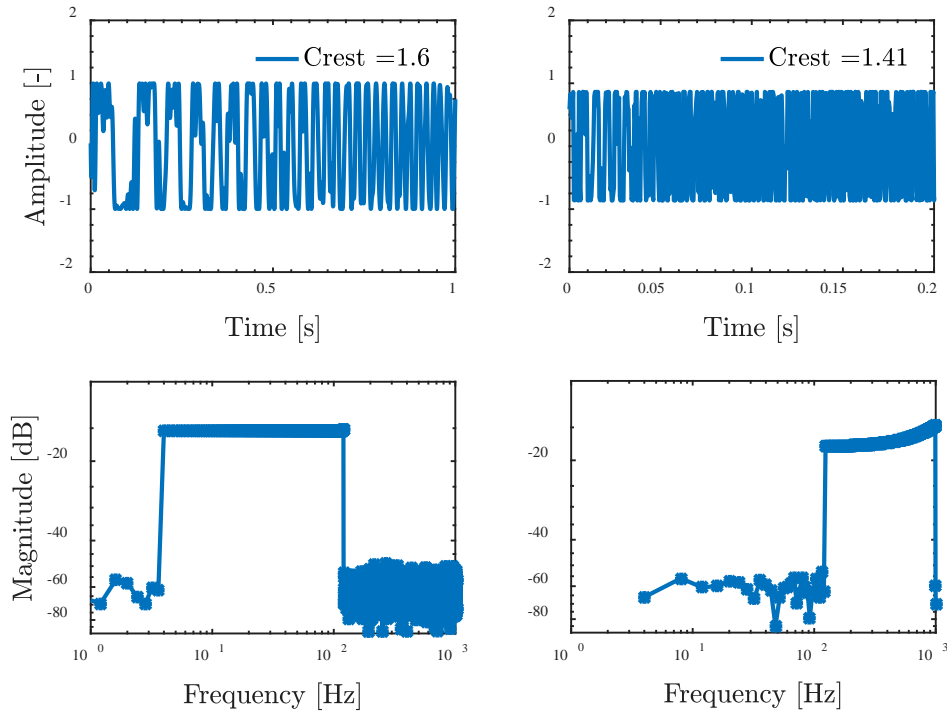
Excitation Design – *Multisine*

- Inner-loop design



1kHz – current control bandwidth
 Weighted Least Square Estimation
Proposed Analyzer

- Inverse Multisine design



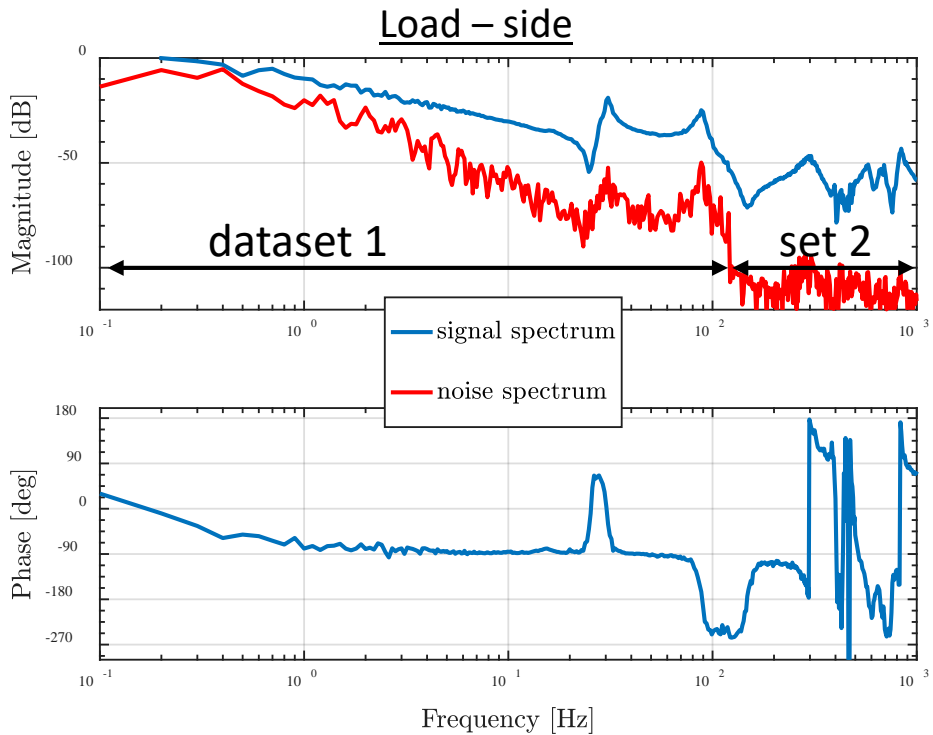
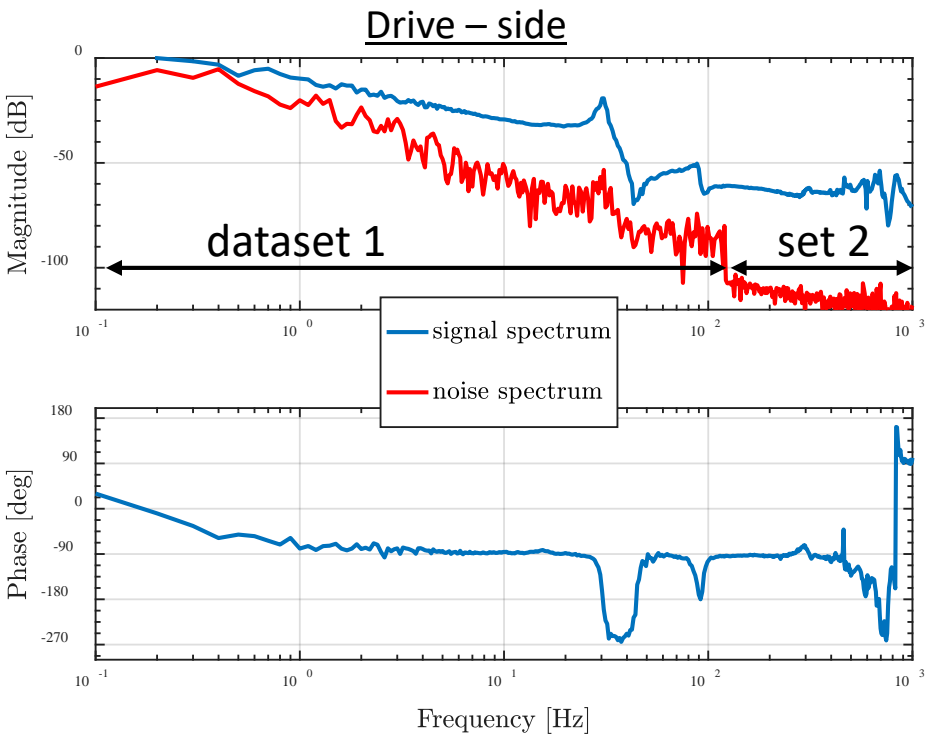
- a) **Quasi-Logarithmic grid**
 wide spectrum – limited frequency lines
- b) **Linear-Segmented grid**
 limited spectrum – high precision data

Non-parametric identification – SIMO

- Maximum likelihood estimator (MLE)

$$\hat{G}_{ML}(j\omega_k) = \frac{\hat{Y}(k)}{\hat{U}(k)} = \frac{M^{-1} \sum_{m=1}^M Y^{(m)}(k)}{M^{-1} \sum_{m=1}^M U^{(m)}(k)}$$

$$\hat{\sigma}_G^2(k) = |\hat{G}_{ML}(j\omega_k)|^2 \cdot \left[\frac{\hat{\sigma}_Y^2(k)}{|\hat{Y}(k)|^2} + \frac{\hat{\sigma}_U^2(k)}{|\hat{U}(k)|^2} - 2\text{Re} \left(\frac{\hat{\sigma}_{YU}^2(k)}{\hat{Y}(k)\overline{\hat{U}(k)}} \right) \right]$$



2 x 10s – segmented experiments

Proposed Analyzer

5.5 EXPERIMENTS

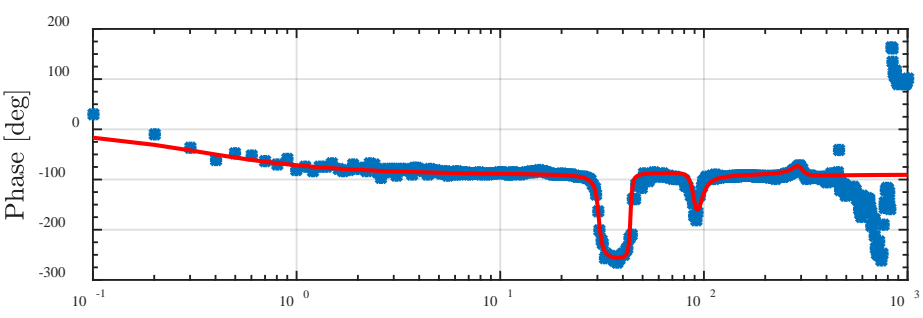
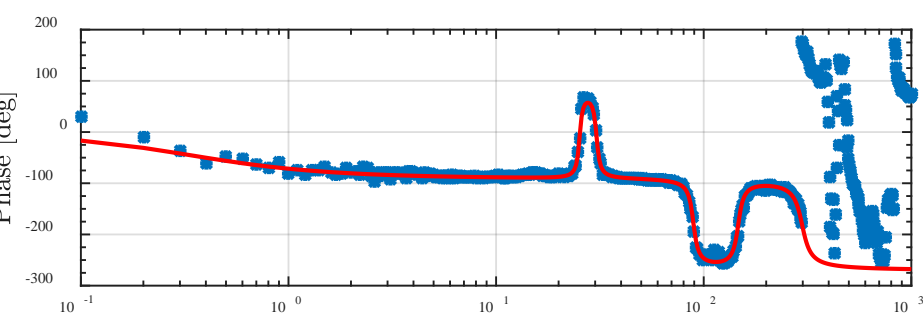
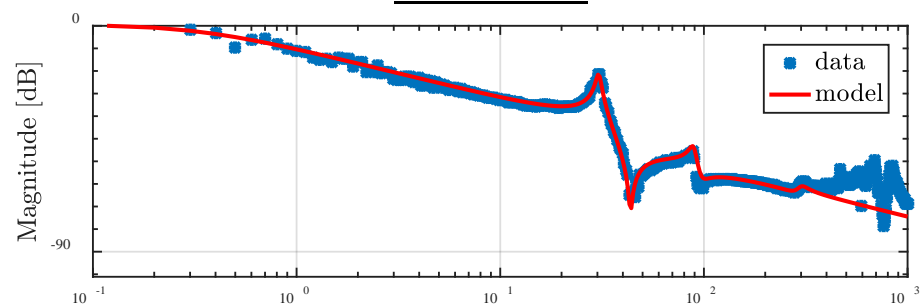
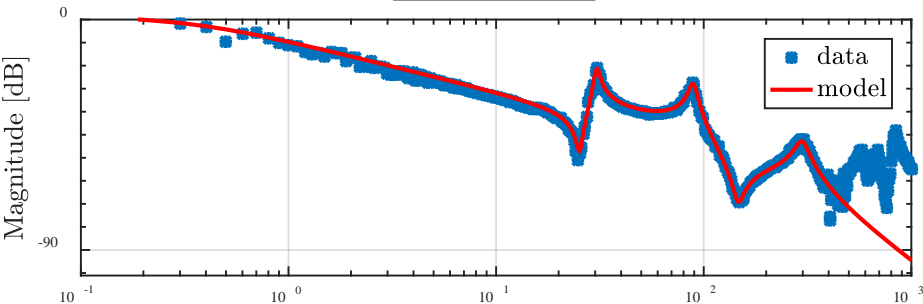
Parametric Identification – SIMO

- Maximum likelihood estimator (MLE)

$$\min_{\theta^{(i)}} \sum_{k=1}^F \frac{\left| \begin{bmatrix} Y_m(s_k) & -U_m(s_k) \end{bmatrix} \begin{bmatrix} D(s_k, \theta) \\ N(s_k, \theta) \end{bmatrix} \right|^2}{\sigma_Y^2(s_k) |D(s_k, \theta)|^2 + \sigma_U^2(s_k) |N(s_k, \theta)|^2 - 2\text{Re}(\sigma_{YU}^2(s_k) D(s_k, \theta) \overline{N(s_k, \theta)})}$$

Drive – side

Load – side



5.24s – SIMO optimization

Proposed Analyzer

FREQUENCY-DOMAIN SYSTEM IDENTIFICATION FOR HIGH-PRECISION CONTROL DESIGN

Thomas Beauduin, Hiroshi Fujimoto (The University of Tokyo)

ご清聴ありがとうございました
Thank you for your kind attention

