

# Gain-scheduling Control of Rotary Inverted Pendulum by Weight Optimization and $H_\infty$ Loop Shaping Procedure

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Gain-scheduling control is one of effective methods for plants whose dynamics changes significantly according to its operating point. A frozen parameter method is known to be a practical gain-scheduling controller synthesis, which interpolates the controllers designed at the prespecified (frozen) operating points according to the current operation point. Hyde et al. proposed a gain-scheduling control that  $H_\infty$  loop shaping procedure is adopted as a controller synthesis at each operating point.  $H_\infty$  loop shaping procedure is based on loop shaping of an open loop characteristic by frequency weights  $W_1(s)$  and  $W_2(s)$ , and is known to be effective for plants with bad condition number.  $W_1(s)$  and  $W_2(s)$  must be chosen so that the shaped plant  $G(s) = W_2(s)P(s)W_1(s)$  has

- (a) an appropriate cut-off frequency,
- (b) a high gain in low frequencies,
- (c) an adequate decay in high frequencies, and
- (d) an appropriate phase property near the gain crossover frequency.

Then the controller  $C_\infty(s)$  is designed for  $G(s)$  maximizing

$$\epsilon \equiv \left( \left\| \begin{bmatrix} I \\ C_\infty \end{bmatrix} (I - GC_\infty)^{-1} \begin{bmatrix} G & I \end{bmatrix} \right\|_\infty \right)^{-1}.$$

However, weight selection satisfying the above control specifications is hard job for a designer.

This paper describes the design of a suboptimal weight and a controller by means of iterative scheme that maximizes the robust stability margin  $\epsilon$  and shapes the open loop characteristic into the desired shape at each of the operating points. Moreover, we formulate a weight optimization problem as a generalized eigenvalue minimization problem, which reduces the designer's burden of weight selection significantly. Fig. 1 shows the design result of the shaped plant  $G(s)$  by applying the proposed weight optimization algorithm in case of the fixed  $W_2(s)$ . According to increase of the iteration number, the open loop characteristic is shaped within the desired region bounded by  $|\bar{s}(j\omega)|$  and  $|\underline{s}(j\omega)|$ . Fig. 2 shows the achieved robust stability margin  $\epsilon$ . The solid and the dashed lines correspond to the case of scheduling both weights and controller and that of scheduling only the controller, respectively. By scheduling both weights and the controller, higher robust stability margin can be ensured, equivalently, higher control performance can be achieved.

Finally, we realize robust and high performance control system by scheduling both weights and controllers. The effectiveness of the proposed control system is verified in terms of the achieved robust stability margin and experimental

time responses of a rotary inverted pendulum which involves strong nonlinear dynamics.

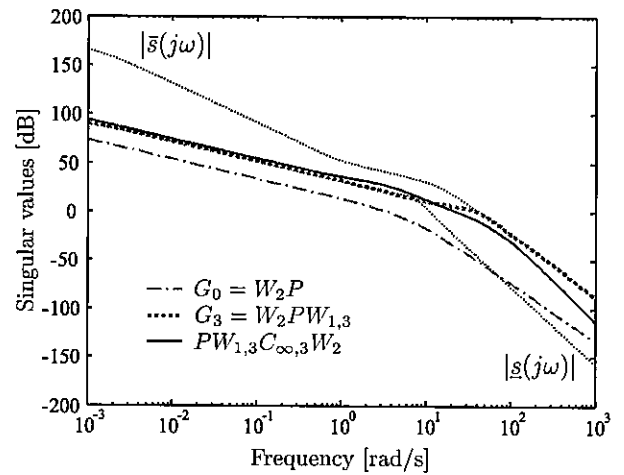


Fig. 1. Singular values of open loop characteristics

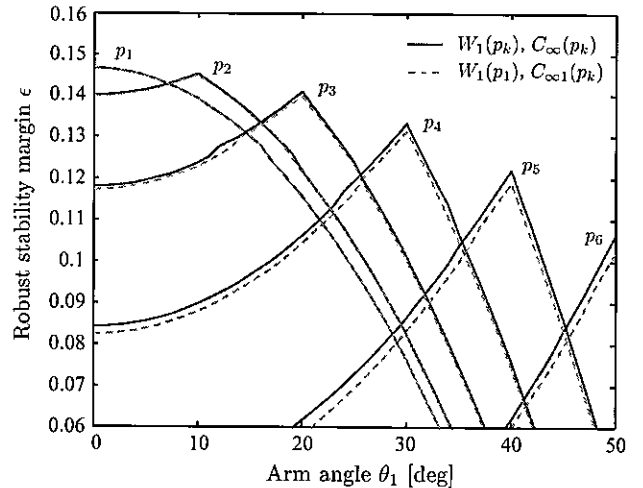


Fig. 2. Robust stability margin  $\epsilon$